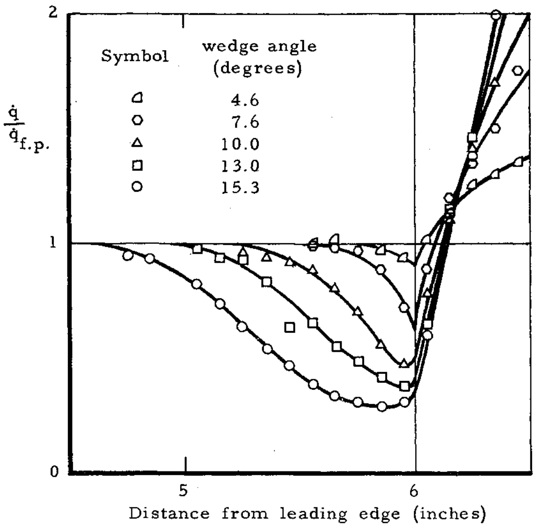

Fig. 2 Heat-transfer distributions for attached and separated flows.

a single shock wave visible at the outer edge of the boundary layer downstream of the corner. The pressure distribution has a single point of inflexion whereas the heat-transfer distribution exhibits a cusp-like minimum at the corner. The decrease in heat transfer ahead of the corner coincides with the initial rise in pressure resulting from the upstream influence of the wedge. Downstream of the corner the heat-transfer rate rises steeply in the region of high pressure gradient, reaching a maximum value just downstream of the peak pressure. As the wedge angle is increased, a small region of flow separation is formed in the corner, as in Fig. 1b, in which two shock waves are visible: one plane oblique shock generated by the boundary-layer separation point upstream of the corner, and a second concave curving shock just downstream of the corner in the reattachment region. The pressure distribution then exhibits a knee just upstream of the corner and the heat transfer is seen to develop a smooth minimum with a continuously changing gradient instead of the cusp typical of the attached flow. As the wedge angle is increased further, as in Figs. 1c and 1d, the size of the separated region grows as the separation point moves upstream, and the knee in the pressure distribution broadens, although the plateau pressure characteristic of fully separated flows is only just attained at the largest wedge angle tested. The heat-transfer minimum retains the smooth curvature and broadens out as the separation point moves upstream. This general trend in heat transfer has previously been observed by Miller, Hijman and Childs,[2] although the detailed structure in large regions of separation appears to differ from the findings of this investigation.

The change in form of the heat-transfer distribution between attached and separated flows is best seen in Fig. 2 where the results obtained for wedge angles from 4.6° to 15.3° are superimposed. From these observations it appears that incipient separation occurs for this case between 7.6° and 10.0° and probably closer to the former wedge angle. An estimate of the wedge angle for incipient separation obtained from an extrapolation of a plot of separation length against wedge angle indicated a value of 8°.

The heat-transfer criterion for the detection of incipient separation described in this note has been used recently by Stollery[3] in a study of wedge separation at $M = 14$. In the current program conducted by the author, the heat-transfer criterion has been used to supplement the evidence of incipient separation obtained from pressure measurements and schlieren photographs However, it is believed that in short running time, low-density hypersonic facilities, the faster response of heat-transfer gages, and the high instrumentation density possible with the thin-film technique will render this criterion for the detection of incipient separation of greater value than the equivalent one based on pressure measurements.

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Hall Potentials in Nonequilibrium MHD Generators

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Introduction

BASED largely on an anticipated need for large quantities of electrical power (megawatt level) for space missions, there is a considerable interest in MHD generators operating at gas temperatures consistent with near future nuclear reactor technology. Since these temperatures are quite low compared to those necessary for thermal ionization of the gas, one must rely on some form of nonequilibrium ionization. Accordingly, experiments[1, 2] have been designed to evaluate the feasibility of creating the desired nonequilibrium state solely by use of the self-induced electric field in the MHD generator. In both of these, it has been demonstrated that such an effect does exist.

At the same time, however, it has been shown that lower Hall voltages than predicted are obtained experimentally. There are several possible explanations for this, all depending on the fact that the plasma is, or is intended to be, in a nonequilibrium state. One explanation suggests that shorting along the segmented electrodes may be causing the low voltages.[3] It will be the purpose of the present note to explore an alternate possibility, which involves shorting along the insulator surfaces.

The essential question to be asked is: Why should one expect the open-circuited Hall voltage to be

$$E_x = \omega\tau(E_y - uB_z) \quad (1)$$

where u is the flow velocity in the x direction, ω is the electron cyclotron frequency, and τ is the mean time between collisions? For the simplest case, where σ is constant and $\mathbf{v} = (u, 0, 0)$ with u constant, the generalized Ohm's law[4] yields Eq. (1). If additional complexity is admitted, velocity profiles may exist so that $\mathbf{v} = (u, v, 0)$ where both may vary in a direction normal to the insulator surfaces, and v is a cross flow. Even in this far more complicated situation it has been found[5] that Eq. (1) is still valid if u is taken to be the average value. This is true in spite of the fact that *axial currents do flow.*

It will be the purpose of the present note to consider further the validity of Eq. (1) when not only velocity profiles are included, but the nonequilibrium effect is also allowed for.

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Table 1 Hall voltages when $\sigma = \sigma_0 + C|j|$, $H_a = 3$

$\omega\tau$	Cu_0B_z	$E_x/\langle u\rangle B_z$	$-\omega\tau$
1	0.5	−1.03	−1.00
10	0.5	−10.02	−10.00
10	0.9	−10.05	−10.00

Analysis

In order to couple the nonequilibrium effect to the flow field in a way simple enough to permit solutions to the flow problem, a one-fluid system of equations has been evolved in which the nonequilibrium effect is accounted for by allowing the electrical conductivity to be a function of the current density.[6]

Two assumptions have been made as to the functional dependency. They are

$$\sigma = \sigma_0 + C|j| \quad (2)$$

$$\sigma = \sigma_R|j|^n \qquad n \geq 0 \quad (3)$$

where the first would seem appropriate when σ is close to σ_0, and the second where σ is considerably changed from its thermal value. Before proceeding to the question at hand it may be of interest to note that, when the foregoing assumptions are combined with the generalized Ohm's law, one finds limiting values that the constants C and n may assume. They are

$$C < 1/|E_y - uB_0| \qquad n < 1 \quad (4)$$

These restrictions are not related to the flow problem but only to the Ohm's law. Further restriction has been found for C by attempting to solve the flow problem for values of C close to but not equal to its value given in Eq. (4).[6] It has been found that no combination of the parameters involved will permit a solution.

Restricting C and n to values below their critical values, the Hartmann flow problem has been considered where it is assumed that nonequilibrium effects exist, $\sigma = \sigma(j)$, and the Hall effect also is present, $\omega\tau \neq 0$. For each case, one obtains a pair of simultaneous, nonlinear, ordinary differential equations with three arbitrary parameters. These parameters, respectively, the axial pressure gradient, the transverse pressure gradient, and the Hall field, are determined iteratively by requiring the average transverse flow to be zero, the average axial current flow to be zero, and the flow velocity at the channel walls to be zero. Solution of these equations, along with the required iterations, has been obtained on an analog computer.

Results

In general, the Hall electric field will be largest when the Faraday electric field L_y is zero. Thus, we will only consider cases for which this assumption has been made. The first set of calculations correspond to the $\sigma(j)$ assumption of Eq. (2), and are presented in Table 1. Additional calculations have been carried out for the $\sigma(j)$ assumption of Eq. (3). In this instance, only the $n = \frac{1}{2}$ case has been evaluated and is presented in Table 2. We recall from Eq. (1) that $E_x/\langle u\rangle B_z = -\omega\tau$ when all nonequilibrium effects are neglected, so that the results as calculated from the present model[6] are compared in the tables to what one would expect from the simple theory.

Table 2 Hall voltages when $\sigma = \sigma_R|j|^{1/2}$, $(\sigma_R/\sigma_0)H_a = 3$

$\omega\tau$	$E_x/\langle u\rangle B_z$	$-\omega\tau$
1	−1.06	−1.00
10	−10.09	−10.00

Discussion

Reviewing our calculated results, we observe that the Hall voltages derived are *larger* than what would have been expected. Also, the larger the Hall effect $\omega\tau$, the smaller this increase, and the closer Cu_0B_z comes to its limiting value the larger the increase becomes. In addition, we note that when the nonequilibrium effect is stronger, $\langle u\rangle$ is larger than when it has been neglected,[6] so that the *dimensional* Hall electric field will be even greater than shown in the tables.

As was noted earlier, when conductivity nonuniformities exist normal to the generator electrodes, one can show that the Hall potential should be reduced either when infinitely finely segmented electrodes are assumed[2] or when finite electrodes are accounted for.[3] By contrast, however, we have shown here that, when such nonuniformities exist normal to the insulators, the Hall voltage is *increased* in spite of the attendant axial current flows. Therefore, one might conclude that the severe reductions predicted by Kerrebrock[3] may be somewhat relieved by the results obtained in the present study.

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A Simple Universal Velocity Profile Equation

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Numerous authors have attempted to fill the need for a continuous universal velocity profile expression, reaching all the way from the wall out into the turbulent flow. These include Miles,[1] van Driest,[2] Szablewski,[3] Reichardt,[4] and Ng.[5] Aside from the fact that none of these precisely represent the others, all are relatively difficult functionally and make hand calculation virtually impossible when one tries to use them to compute shear stress from velocity data. To provide a simpler expression for such work, we have used the expression

$$Y^+ = U^+ + (U^+/a)^n = U^+ + (U^+/8.74)^7 \quad (1)$$

Here $U^+ \equiv u(\rho/\tau)^{1/2}$ and $Y^+ \equiv (y/\nu)(\tau/\rho)^{1/2}$, where u is

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